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INFLUENCE OF HEAT SPREAD IN THE MEASUREMENT LAYER ON THE ACCURACY IN MEASURING LOCAL HEAT FLUXES BY THE GRADIENT METHOD

É. N. Voznesenskii and V. I. Nemchenko

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INTRODUCTION

One of the most widely known methods of measuring heat flux is the steady gradient method, or the method of an auxiliary wall [1]. But this method, despite its high sensitivity and relative simplicity, is used relatively rarely in model gas-dynamic experiments. Data are practically absent on the use of the method of an auxiliary wall in heat-transfer research carried out on models in low-density aerodynamic installations. The main reasons preventing its extensive use in measurements of such a kind are the inadequate spatial resolution of individual calorimeters at the relatively small dimensions of the models themselves and of the regions of action of the gas stream, as well as the appearance of temperature "steps" at the surface of the test model at the calorimeter mounting points in a number of cases. These drawbacks are eliminated to a considerable extent, however, when the method of an auxiliary wall is carried out in the installation proposed in [2] for bodies having a plane or slightly curved surface. The idea of the proposed realization consists in the following. The body of the model, thermostatically controlled in some way, is covered by a layer of material of low thermal conductivity (the measurement layer) so thin that the heat spread in it becomes insignificant at the places most important from the aspect of heat transfer. Then the local temperature drop between opposite sides of the layer, when the thermal conductivity $\lambda =$ const of the material is known, is proportional to the local value of the incident heat flux, and the problem is reduced to recording this temperature drop with the required accuracy. The distribution of the temperature difference between the surfaces of the measurement layer is measured with differential microthermocouples, the hot junctions of which are placed on the outer surface of the layer in a certain way which allows for the heat outflow along the thermocouple leads [2].

Consequently, in the realization of the method of an auxiliary wall in the given variant an important point, along with the question of the correct placement of the differential thermocouple junctions discussed earlier in [2], is the allowance for the influence of tangential heat fluxes in the measurement layer and the determination of the error introduced by these fluxes into the measurement result. A correct estimate of the latter would make it possible to select the required thickness of the measurement layer corresponding to the conditions of a specific experiment. The effects of heat spread have been analyzed earlier mainly for nonsteady methods of measuring heat fluxes [3, 4]. The problem of heat spread has evidently not been raised in general form in application to the method of an auxiliary wall. Only individual particular examples are known [1, 2].

In the present work an attempt is made to derive, from sufficiently general premises, approximate functions connecting the error in the measurement of the heat-flux distribution due to heat spread in the measurement layer with the parameters of the layer and the characteristics of the quantity being measured. The suitability of the equations obtained for prac-

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tical use is tested by comparing the approximate errors calculated from these equations with the exact errors for a number of model problems of heat conduction admitting of an analytical solution, as well as with experiment.

In the specific installation of [2] a thin measurement layer was glued to a thermostatically controlled body. In addition, in some cases a special protective film can be deposited on the outer surface of the measurement layer to protect the thermocouple junctions and eliminate hydrodynamic perturbations. Thus, in setting up the problem of heat spread for the method of an auxiliary wall one must keep in mind that a real covering is multilayered, generally speaking.

§1. With allowance for the remarks which were made, let us consider the steady-state problem of heat conduction in a thin-layered composite covering deposited onto a plane or slightly curved isothermal surface and consisting of three uniform layers. The thermal contact between all the layers is taken as ideal. We designate the thickness and coefficient of thermal conductivity of the inner layer (layer 1) as h_1 and λ_1, respectively, the same parameters for the central layer (layer 2), filling the role of the measurement layer, as h_2 and λ_2, and those for the outer layer (layer 3) as h_3 and λ_3. We take the orthogonal coordinate system in the usual way: the x and y coordinates along the isothermal surface and the z coordinate along the normal to it.

Then the temperature field within each layer can be described by a Laplace equation in the form

$$\Delta T = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = 0. \tag{1.1}$$

Let a heat flux $q_w(x, y)$ fall on the outer surface of the covering, with the function $q_w(x, y)$ being continuous. The boundary conditions of the problem and the conditions for joining the solutions at the transitions from layer 1 to layer 2 and from layer 2 to layer 3 are written in the form

$$z = 0,\ T(x, y, 0) = T_0 = \text{const}; \tag{1.2}$$

$$z = h_1,\ T(x, y, h_1 - 0) = T(x, y, h_1 + 0) = T_1, \tag{1.3}$$

$$\begin{gathered} \lambda_1 \frac{\partial T}{\partial z}\Big|_{h_1 - 0} = \lambda_2 \frac{\partial T}{\partial z}\Big|_{h_1 + 0}; \\ z = h_{12},\ T(x, y, h_{12} - 0) = T(x, y, h_{12} + 0) = T_2, \\ \lambda_2 \frac{\partial T}{\partial z}\Big|_{h_{12} - 0} = \lambda_3 \frac{\partial T}{\partial z}\Big|_{h_{12} + 0}; \end{gathered} \tag{1.4}$$

$$z = h_{123},\ \lambda_3 \frac{\partial T}{\partial z}\Big|_{h_{123}} = q_w(x, y), \tag{1.5}$$

where $h_{12} = h_1 + h_2$; $h_{123} = h_1 + h_2 + h_3$. The fact that the directions of the Oz axis and the active heat flux are opposite is taken into account in (1.5). Integration of (1.1) across layers 2 and 3 leads to the following equations:

$$\lambda_3 \int_{h_{12}}^{h_{123}} \Delta' T d\zeta + \lambda_3 \frac{\partial T}{\partial z}\Big|_{h_{123}} - \lambda_3 \frac{\partial T}{\partial z}\Big|_{h_{12}+0} = 0; \tag{1.6}$$

$$\lambda_2 \int_{h_1}^{h_{12}} \Delta' T d\zeta + \lambda_2 \frac{\partial T}{\partial z}\Big|_{h_{12}-0} - \lambda_2 \frac{\partial T}{\partial z}\Big|_{h_1+0} = 0; \tag{1.7}$$

$$\frac{\lambda_2}{h_2} \int_{h_1}^{h_{12}} dz \int_{h_1}^{z} \Delta' T d\zeta + \lambda_2 \frac{T_2 - T_1}{h_2} - \lambda_2 \frac{\partial T}{\partial z}\Big|_{h_1+0} = 0, \tag{1.8}$$

where $\Delta' = \partial^2/\partial x^2 + \partial^2/\partial y^2$. From (1.4)-(1.8) one can obtain an exact expression for the error ε due to spreading:

$$\varepsilon = \frac{1}{q_w(x, y)} \left[\lambda_2 \frac{T_2 - T_1}{h_2} - q_w(x, y) \right] = \frac{1}{q_w(x, y)} \left[\lambda_3 \int_{h_{12}}^{h_{123}} \Delta' T dz - \lambda_2 \int_{h_1}^{h_{12}} \Delta' T dz - \frac{\lambda_2}{h_2} \int_{h_1}^{h_{12}} dz \int_{h_1}^{z} \Delta' T d\zeta \right]. \tag{1.9}$$

Let the total thickness h_{123} of the covering be such that the heat spread in it is small everywhere in the region under consideration (i.e., $\varepsilon \ll 1$). This means that the temperature distribution over the z coordinate within each layer is nearly linear, and, moreover, by virtue of (1.2)-(1.4),

$$\lambda_1 \frac{T_1 - T_0}{h_1} \approx \lambda_2 \frac{T_2 - T_1}{h_2} \approx \lambda_3 \frac{T_3 - T_2}{h_3},$$

where $T_3 = T(x, y, h_{123})$. Evaluating the integrals in Eq. (1.9) on the basis of these considerations, we obtain a sufficiently simple approximate expression for the value,

$$\varepsilon \approx \frac{\lambda_2 h_2}{3 q_w(x, y)} \left[1 + 3\frac{\lambda_3 h_3}{\lambda_2 h_2} + \frac{3}{2}\frac{\lambda_2 h_1}{\lambda_1 h_2} + \frac{3}{2}\left(\frac{h_3}{h_2}\right)^2 + 3\frac{\lambda_3}{\lambda_1}\frac{h_3 h_1}{h_2^2}\right] \times$$

$$\times \Delta'(T_2 - T_1) \approx \frac{h_2^2}{3}\left[1 + 3\frac{\lambda_3 h_3}{\lambda_2 h_2} + \frac{3}{2}\frac{\lambda_2 h_1}{\lambda_1 h_2} + \frac{3}{2}\left(\frac{h_3}{h_2}\right)^2 + 3\frac{\lambda_3}{\lambda_1}\frac{h_3 h_1}{h_2^2}\right]\frac{\Delta'(T_2 - T_1)}{T_2 - T_1}. \tag{1.10}$$

The functions obtained make it possible to trace the influence of various parameters of the covering on the degree of heat spread and thereby to estimate the thicknesses of the protective, glue, and measurement layers assuring a possible error satisfying the demands of a specific investigation.

Thus, in order to neglect the heat spread due to the presence of the glue and protective layers one must satisfy the conditions

$$\lambda_3 h_3 \ll \lambda_2 h_2, \; \lambda_2 / h_2 \ll \lambda_1 / h_1, \; h_3 \ll h_2.$$

Then

$$\varepsilon \approx \frac{\lambda_2 h_2}{3}\frac{\Delta'(T_2 - T_1)}{q_w(x, y)} \approx \frac{h_2^2}{3}\frac{\Delta'(T_2 - T_1)}{T_2 - T_1}. \tag{1.11}$$

In those cases when the protective layer is absent but the thickness of the glue layer is comparable with the thickness of the measurement layer, the error due to spreading is estimated from the equation

$$\varepsilon \approx \frac{\lambda_2 h_2}{3 q_w(x, y)}\left(1 + \frac{3}{2}\frac{\lambda_2 h_1}{\lambda_1 h_2}\right)\Delta'(T_2 - T_1) \approx \frac{h_2^2}{3}\left(1 + \frac{3}{2}\frac{\lambda_2 h_1}{\lambda_1 h_2}\right)\frac{\Delta'(T_2 - T_1)}{T_2 - T_1}. \tag{1.12}$$

Equations (1.10)-(1.12), depending on the relations between the parameters of the layer comprising the covering, can be used for an approximate calculation of the error on the basis of measurement results. Below we use only Eq. (1.11) and the consequences following from it, as corresponding most closely to the properties of a real covering [2]. If the function $q_w(x, y)$ is twice differentiable, the unknown error can be calculated approximately from the following equation, equivalent to (1.11):

$$\varepsilon \approx \frac{h_2^2}{3}\frac{\Delta' q_w(x, y)}{q_w(x, y)}. \tag{1.13}$$

This equation is convenient to use for comparing approximate errors due to heat spread with exact errors obtained from the solution of heat-conduction problems.

The equations presented (including those pertaining to a multilayer covering) remain valid everywhere for a layer of finite size having a thermally insulated side surface. With side boundary conditions of a different character their use is admissible for sections of the measurement layer sufficiently remote from its edges. In practice, this remoteness comprises five thicknesses of the measurement layer [5].

A general result following from Eq. (1.11) or (1.13) is important for practical purposes: When the thickness of the measurement layer is very small, the error due to spreading is proportional to the square of the thickness of this layer and has the order of magnitude

$$\varepsilon \sim (h_2/L_*)^2, \tag{1.14}$$

where L_* is the characteristic geometrical size of the nonuniformity of the active heat flux.

This is a major circumstance for the selection of the thickness of the measurement layer in the construction of a model.

§2. To test the correctness of the estimates given above for the error of the method due to heat spread, let us turn to two-dimensional problems of heat conduction in uniform flat plates having an analytical solution, with boundary conditions corresponding to the working conditions of the measurement layer. First we consider the plane problem

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial z^2} = 0,$$
$$T(x, 0) = T_1 = \text{const}, \quad \left.\frac{\partial T}{\partial x}\right|_{x=0} = \left.\frac{\partial T}{\partial x}\right|_{x=L} = 0, \quad \lambda \left.\frac{\partial T}{\partial z}\right|_{z=h} = q_w\left(\frac{x}{L}\right) \tag{2.1}$$

(h and L are the thickness and length of the plate, respectively). From the solution of this mixed boundary-value problem, which can be obtained by the method of separation of variables, we obtain the following equation for the quantity $\lambda(T_2 - T_1)/h = \lambda\Delta T_w/h$:

$$\lambda \frac{\Delta T_w}{h} = \int_0^1 q_w(\xi)\, d\xi + 2\sum_{n=1}^{\infty} \frac{\operatorname{th} \pi n h^0}{\pi n h^0} \cos \pi n x^0 \int_0^1 q_w(\xi) \cos \pi n \xi d\xi, \tag{2.2}$$

where $x^0 = x/L$; $h^0 = h/L$; $T_2 = T(x, h)$; $\Delta T_w = T_2 - T_1$, and L is the length of the plate. Numerical calculations of the series of (2.2) were made on a computer for the function

$$q_w(x^0) = (Ax^0/2)^2 \exp(-Ax^0 + 2) \tag{2.3}$$

with A = 20, 30, and 40 and h^0 = 0.01, 0.02, 0.03, 0.04, and 0.05.†

The exact errors ε_e on the basis of the results of these calculations are presented in Fig. 1a, b for A = 20, and 40, respectively. It is seen, first of all, that a decrease in the thickness of the plate leads to convergence of the values of $\lambda\Delta T_w/h$ and $q_w(x^0)$ at all x^0 and, secondly, that the smoother the function $q_w(x^0)$ (the smaller the value of the parameter A), the faster $\lambda\Delta T_w/h$ converges to $q_w(x^0)$. This means that with a decrease in the thickness of the plate the transverse temperature profiles in it approach linear profiles, and hence a tendency toward convergence of the approximate and exact errors ε_{ap} and ε_e must occur.

Such a relationship, confirming the correctness of the error estimates derived in Sec. 1, is actually observed. Table 1, which contains the data of a calculation of the errors for a series of values of x^0 from the vicinity of the maximum $q_w(x^0) = q_{max}$, which is the most interesting in applications, as a rule, serves as an illustration of this. The values of ε_{ap} are calculated from Eq. (1.13). A determination of the order of magnitude of ε from Eq. (1.14) indicates that it coincides with the actual values in the entire range of h^0 for the vicinity under consideration. The smallest of the distances from the point of the maximum value of $q_w(x^0)$ to the point at which $q_w(x^0) = 0.1q_{max}$ is taken as the characteristic size L_*.

Now let us turn to the axisymmetric case. The statement of the problem is analogous to (2.1):

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} = 0,$$
$$T(r, 0) = T_1 = \text{const}, \quad \left.\frac{\partial T}{\partial r}\right|_{r=R} = 0, \quad \lambda \left.\frac{\partial T}{\partial z}\right|_{z=h} = q_w\left(\frac{r}{R}\right). \tag{2.4}$$

Here h and R are the thickness and radius of the flat disk, respectively. As an example of a heat-flux distribution function we take the function

$$q_w(r^0) = \frac{1}{1.4028}[0.4028 + J_0(3.8317 r^0)], \tag{2.5}$$

where $J_0(x)$ is a zero-order Bessel function of the first kind; $r^0 = r/R$. Solving the problem (2.4) with allowance for (2.5), we obtain

†The calculations were made by V. A. Buzhinskii.

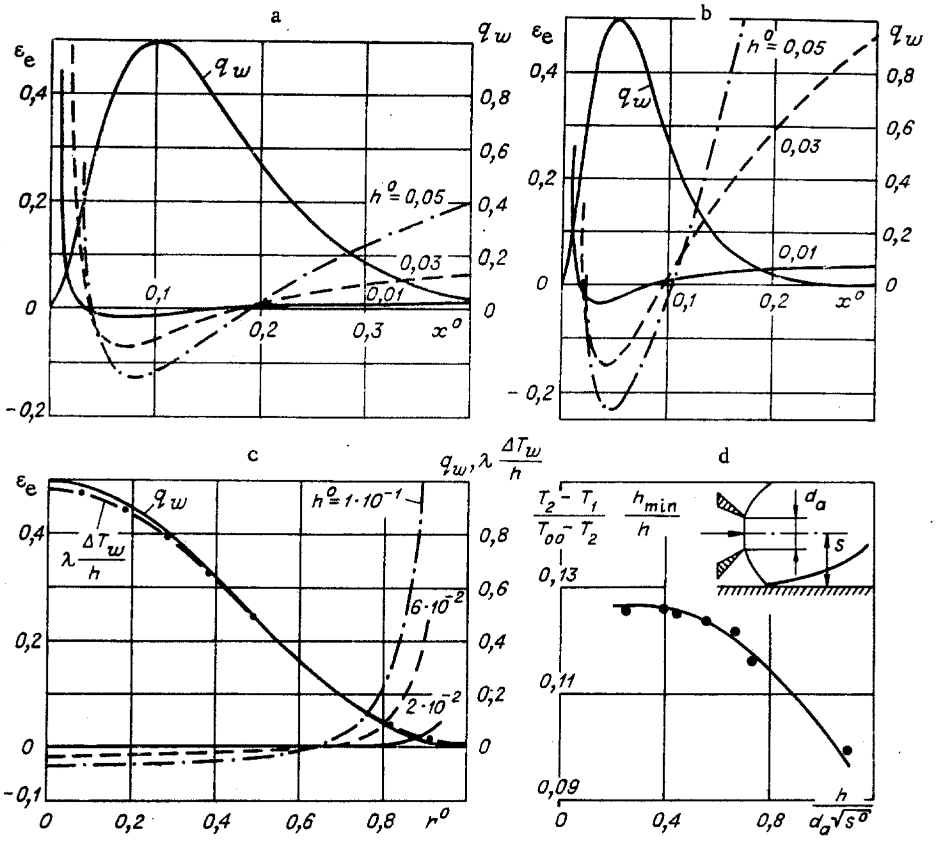

Fig. 1

TABLE 1

	A=20				A=40			
x^0	$-\varepsilon_e$	$-\varepsilon_{ap}$	$\varepsilon_{ap}/\varepsilon_e$	h^0	$-\varepsilon_e$	$-\varepsilon_{ap}$	$\varepsilon_{ap}/\varepsilon_e$	x^0
				$1\cdot10^{-2}$	0,0330	0,0504	1,527	
	0,0327	0,0533	1,630	$2\cdot10^{-2}$	0,0897	0,2015	2,246	
0,05	0,0530	0,1200	2,264	$3\cdot10^{-2}$	0,1238	0,4543	3,662	0,03
	0,0645	0,2133	3,307	$4\cdot10^{-2}$	0,1455	0,8059	5,539	
	0,0701	0,3333	4,755	$5\cdot10^{-2}$	0,1638	1,2592	7,687	
				$1\cdot10^{-2}$	0,0336	0,0383	1,140	
	0,0390	0,0446	1,144	$2\cdot10^{-2}$	0,0966	0,1533	1,587	
0,07	0,0632	0,1004	1,589	$3\cdot10^{-2}$	0,1504	0,3450	2,294	0,04
	0,0892	0,1786	2,002	$4\cdot10^{-2}$	0,1910	0,6133	3,211	
	0,1116	0,2790	2,500	$5\cdot10^{-2}$	0,2233	0,9582	4,291	
				$1\cdot10^{-2}$	0,0310	0,0267	0,861	
	0,0267	0,0267	1,000	$2\cdot10^{-2}$	0,0883	0,1067	1,208	
0,10	0,0566	0,0600	1,060	$3\cdot10^{-2}$	0,1449	0,2400	1,656	0,05
	0,0887	0,1067	1,203	$4\cdot10^{-2}$	0,1919	0,4267	2,224	
	0,1201	0,1667	1,388	$5\cdot10^{-2}$	0,2294	0,6668	2,907	

TABLE 2

r^0	h^0	ε_e	ε_{ap}	$\varepsilon_{ap}/\varepsilon_e$
0	$2\cdot10^{-2}$	$-1,3922\cdot10^{-3}$	$-1,3955\cdot10^{-3}$	1,0024
	$6\cdot10^{-2}$	$-1,2299\cdot10^{-2}$	$-1,2559\cdot10^{-2}$	1,0212
	$1\cdot10^{-1}$	$-3,2950\cdot10^{-2}$	$-3,4887\cdot10^{-2}$	1,0588
0,261	$2\cdot10^{-2}$	$-1,2791\cdot10^{-3}$	$-1,2825\cdot10^{-3}$	1,0026
	$6\cdot10^{-2}$	$-1,1303\cdot10^{-2}$	$-1,1542\cdot10^{-2}$	1,0212
	$1\cdot10^{-1}$	$-2,9924\cdot10^{-2}$	$-3,2062\cdot10^{-2}$	1,0715
0,496	$2\cdot10^{-2}$	$-0,8032\cdot10^{-3}$	$-0,8050\cdot10^{-3}$	1,0032
	$6\cdot10^{-2}$	$-0,7094\cdot10^{-2}$	$-0,7252\cdot10^{-2}$	1,0223
	$1\cdot10^{-1}$	$-1,9020\cdot10^{-2}$	$-2,0145\cdot10^{-2}$	1,0592
0,757	$2\cdot10^{-2}$	$2,4519\cdot10^{-3}$	$2,4599\cdot10^{-3}$	1,0032
	$6\cdot10^{-2}$	$2,1677\cdot10^{-2}$	$2,2139\cdot10^{-2}$	1,0213
	$1\cdot10^{-1}$	$5,8084\cdot10^{-2}$	$6,1510\cdot10^{-2}$	1,0590

$$\lambda\frac{\Delta T_w}{h}=\frac{1}{1.4028}\left[0.4028+\frac{\operatorname{th}3.8317h^0}{3.8317h^0}J_0(3.8317r^0)\right] \quad (2.6)$$
$$(h^0=h/R;\ T_2=T(r^0,h^0);\ \Delta T_w=T_2-T_1).$$

The results of a calculation of ε_e from (2.5) and (2.6) for $h^0 = 2\cdot10^{-2}$, $6\cdot10^{-2}$, and $1\cdot10^{-1}$ and of $q_w(r^0)$ and $\lambda\Delta T_w/h$ for $h^0 = 1\cdot10^{-1}$ are shown in Fig. 1c. The approximate and exact values of the error due to heat spread and their comparison are given in Table 2.

One can see that the properties of the behavior of the approximate and exact values of the error noted in the discussion of the plane problem are also retained here, with considerably better agreement between the data of the approximate and exact calculations occurring in the latter case. The result obtained is not accidental and is easily explainable from the standpoint of Sec. 1. Actually, in the plane problem the function (2.5), in contrast to (2.3), is sufficiently smooth, since $L_* \sim R$ for it and $h/L_* \ll 1$ in all cases, whereas for the function (2.3), $AL_* \ll L$ for the chosen values of the parameter and $h/L_* \leq 1$ in the majority of the variants.

An experimental study of the influence of heat spread in the measurement layer on the accuracy in measuring the heat flux was carried out in a vacuum wind tunnel in application to the problem of the interaction of a strongly underexpanded jet of heated gas with a flat barrier. It is known that in this case a heat-flux distribution distinguished by a large nonuniformity is realized at the surface of the barrier with a characteristic maximum being present [6].

The barrier model consisted of a hollow copper body, with inlets for the thermostatic-control liquid, to which interchangeable plastic plates from $1\cdot10^{-3}$ to $4.4\cdot10^{-3}$ m thick were glued with epoxy resin. The thickness of the resin layer (its thermal conductivity is close to the thermal conductivity of plastic [1]) was about $1\cdot10^{-4}$ m. The plastic plates, playing the role of the measurement layer, were equipped with differential and absolute (to determine the temperature of the inner surface of the layer) microthermocouples. The model was set up in the tunnel chamber with the working plane parallel to the axis of the nozzle — the jet source — at a distance $s^0 = s/d_a = 6$ from it, where d_a and s are the dimensional diameter of the exit cross section of the nozzle (in the experiments $d_a = 1.65\cdot10^{-3}$ m) and the dimensional distance from the nozzle axis to the surface of the model, respectively. In the course of the experiments the required thickness of the measurement layer was sought by the method of successive approximations.

The results of an analysis of the experiments for the point q_{max} are presented in Fig. 1d, for one of the modes of tunnel operation [2]. The dimensionless thickness of the measurement layer is laid out along the abscissa, while a value proportional to the ratio of the measured and actual heat fluxes is laid out along the ordinate ($T_{0\infty}$ is the stagnation temper-

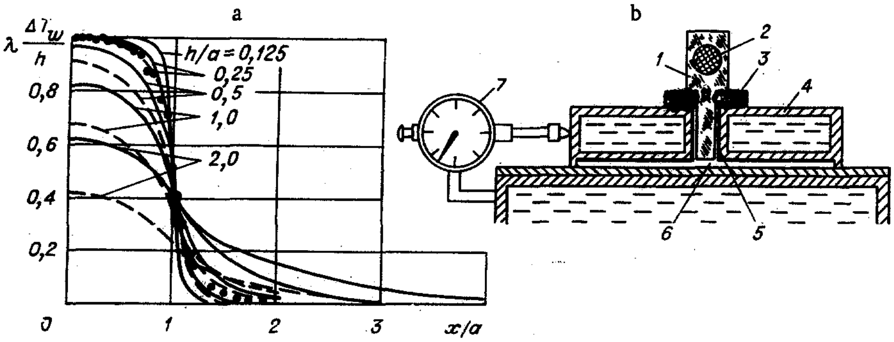

Fig. 2

ature of the jet and h_{min} is the least thickness of the plastic plate out of those used in the experiments). The curve plotted from the test data not only clearly shows the weakening of the influence of heat spread in the measurement layer with a decrease in its thickness, but it testifies in favor of the equations of Sec. 1 at the same time. For example, an estimate of the error due to heat spread on the basis of Eq. (1.14) using data from [2] on the distributions of q_w over a model with a plastic measurement layer $0.94 \cdot 10^{-3}$ m thick gives a value on the order of 1.5% for it, which agrees fully with the behavior of the curve in Fig. 1d, near the corresponding value of the coordinate $h/d_a\sqrt{s^0} = 0.247$.

§3. Cases of discontinuous heat-flux distributions are particularly interesting. Heat spread is always significant at points of discontinuity (we are talking about discontinuities such that the jump in the heat flux is comparable with the arithmetic mean of the heat fluxes at the edges of the discontinuity), and it is important to know the dimensions of that vicinity outside of which the influence of the discontinuity can be neglected. In order to isolate this phenomenon in pure form, we again consider the steady-state problem of heat conduction for a plate in the plane (2.1) and symmetric (2.4) statements with a heat flux at the surface of the plate in the form of a step function,

$$q_w(x),\ \ q_w(r)=\begin{cases}1 & \text{at}\quad 0<x<a,\ \ 0\leqslant r<a,\\ 0 & \text{at}\quad a<x\leqslant L,\ \ a<r\leqslant R,\end{cases}$$

where a is a parameter determining the geometrical dimensions of the region of action of the discontinuous heat-flux distribution. The results of its solution* and an experimental test of one of the variants ($h/a = 0.24$) for the plane case are presented in Fig. 2a. There is a diagram of the experiment in Fig. 2b. A heat flux of almost Π-shaped form was created by the thermal radiation of the blackened lower face of the irregularly shaped body 1, made of copper and equipped with a resistance heater 2. The position of this body was fixed with the help of a thermally insulating gasket 3 inside the slot of the cooled platform 4 in such a way that the emitting face of the body was set flush with the lower surface of the platform. The walls of the slot were coated with a reflecting Mylar film 5 with a coefficient of reflection of 0.95-0.98.

The platform was moved along the blackened surface of the plastic measurement layer. The movement of the platform relative to the monitoring differential microthermocouple 6 was determined with a micrometer 7. The distance between the plane of the emitting face and the surface of the model was $0.3 \cdot 10^{-3}$ m, the dimensions of the emitting face were $7.9 \cdot 10^{-3}$ m × $70 \cdot 10^{-3}$ m, and the thickness of the plastic layer was $0.95 \cdot 10^{-3}$ m. The experiment was carried out under vacuum conditions. The agreement of its results with the calculation is fully satisfactory (points in Fig. 2a). A certain discrepancy observed outside the zone of action of the heat flux is evidently explained by the unavoidable departure of the form of the realized heat flux from a Π shape. The results of calculations for a plate of finite size with $h/L = 2 \cdot 10^{-2}$ (solid curves) in the plane case and $h/R = 1 \cdot 10^{-2}$ (dashed curves) in the axisymmetric case are presented in Fig. 2a. Calculations of the plane problem for a plate of infinite size (the value of h is fixed) were made in addition. Their results coincide with those presented in the first three to four significant figures. There is no doubt that such correspondence also occurs in the axisymmetric problem.

*The computer calculations of this problem were made by V. E. Isaev.

In comparing the solutions of the plane and axisymmetric problems we note that with an increase in the parameter a (i.e., with an increase in the characteristic spatial dimension of the heat flux relative to the thickness of the measurement layer) the difference between them disappears. For $h/a = 0.125$, in particular, the two solutions coincide with an accuracy of about 2% when $x, r < a$.

Finally, on the basis of an analysis of the calculations performed, one can formulate the following general judgement about the influence of discontinuities of heat flux on the degree of heat spread in a measurement layer: The error caused by a discontinuity in heat flux becomes less than 0.01 of the size of this discontinuity in absolute value at distances $\Delta x, \Delta r \geq 3h$ from the point of the discontinuity, and hence they can be ignored in practical estimates. Thus, with the appropriate choice of the thickness of the measurement layer one can measure discontinuous heat-flux distributions, with satisfactory accuracy for practical work, everywhere except for the rather small vicinity of each discontinuity determined above.

In conclusion, we present as an example the results of a measurement of the heat-flux distributions on a flat barrier with a strongly underexpanded jet acting on it, obtained by the method of an auxiliary wall in a vacuum wind tunnel. A model with a plastic measurement layer $0.95 \cdot 10^{-3}$ m thick and the measurement system described in [2] were used. The flat barrier was placed in the field of flow of a strongly underexpanded air jet with a Mach number $M_{a_{geom}} = 3.25$ at the nozzle cut without allowance for the boundary layer. The diameter of the nozzle exit cross section was $d_a = 3.66 \cdot 10^{-3}$ m. The pressure and stagnation temperature of the jet were $p_0 = 7 \cdot 10^4 - 7.2 \cdot 10^4$ Pa and $T_{00} = 400$°K, respectively, the range of values of the temperature factor was $\bar{T}_w = T_2/T_{00} = 0.72-0.81$, the pressure of the working chamber outside the jet was $p_\infty = 1.3 \cdot 10^1$ Pa, $\alpha = 0$ and 30°, and $s^0 = 2$ and 4 ($s^0 = s/d_a$ and α is the an-

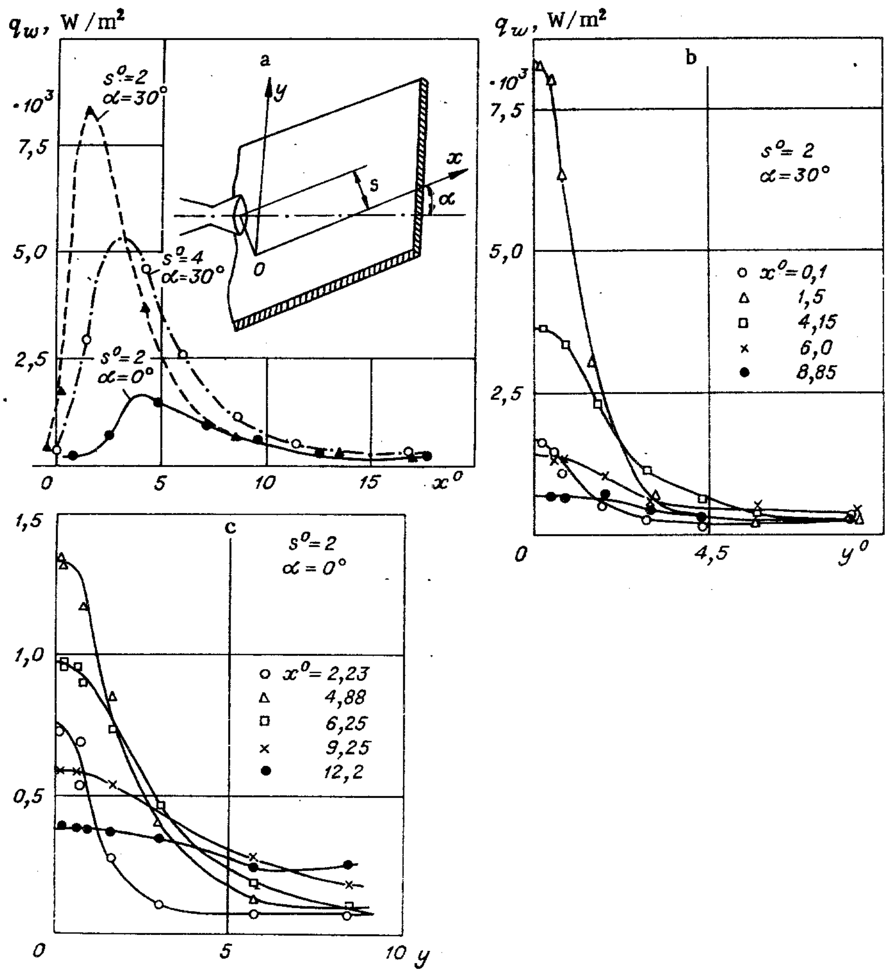

Fig. 3

gle between the nozzle axis and the working surface of the model). The heat-load distributions obtained for these conditions are shown in Fig. 3a-c. All the values of the coordinates x and y and the distances s in these graphs are normalized to the value d_a. The presence of a maximum of q_w, about which we spoke earlier, is characteristic of all the distributions presented. Its height grows both with an increase in α and with a decrease in s°; as s° increases the maximum of q_w shifts downstream (see Fig. 3a). The maxima of the longitudinal side distributions of q_w (along lines of y° = const) shift downstream with an increase in y° (see Fig. 3b, c). Thus, the same qualitative properties are inherent to heat-flux distributions on a flat surface as to distributions of force loads [7].

Estimation calculations of the error due to heat spread in accordance with (1.14) give a value on the order of 1% for all the cases presented. The total error of heat-flux measurements comprises about 7% in the present experiments (owing to errors in the calibration of the microthermocouples and in the determination of the coefficient of thermal conductivity of the material of the measurement layer).

The results presented above show that the method of an auxiliary wall, in the statement proposed in [2], allows one to measure, with good accuracy and high localizability, the distributions of small heat fluxes over models in low-density wind tunnels.

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